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Given Data of System Requirements for Canonical Controller Design

Yuki Ochiai[1], Triet Nguyen-Van[2] and Shin Kawai[3]

Abstract— **This study aims to clarify the requirements for designing a canonical controller using data obtained from unknown systems. Based on Willems' fundamental lemma, this study discusses the conditions under which the system's data can achieve the desired behavior, and then clarifies the data requirements for a canonical controller. The clarified requirements provide sufficient conditions for designing a canonical controller using finite-length data. Furthermore, simulations were conducted to examine the practicality of the clarified requirements. The simulation results demonstrate that adding disturbances to the reference input can extend the finite interval that satisfies the design requirements for a canonical controller.**

I. Introduction

Data-driven control, a control method that uses only the data obtained from the system, has received considerable attention in recent years. Unlike model-based control, data-driven control does not require mathematical models. Therefore, it can reduce the time and financial costs associated with modeling and designing control systems for plants that are difficult to model. Additionally, it is easy to readjust existing control systems based on data in response to factors such as the aging of industrial products. For these reasons, this method is being applied to a wide range of real-world applications, and the technology is expected to further develop and expand its scope of application in the future.

Despite these advantages, the challenge of guaranteeing stability remains a significant hurdle for data-driven control. While model-based control guarantees stability through a mathematical model, data-driven control relies only on data for system design, making stability guarantee more difficult. Guaranteeing stability is critical for the safe operation of systems and is a crucial issue in control engineering. Persis and Tesi discussed stability using input-output data and state variables [1]. However, stability has yet to be guaranteed using only input-output data. Given the nature of data-driven control, which relies entirely on data for system design, a method for guaranteeing stability based only on input-output data, without state variables, is desirable.

One approach to guaranteeing the stability of data-driven control is the design of canonical controllers using data. A canonical controller is a controller that matches the behavior of the system with the desired behavior. The term "behavior" refers to the set of time trajectories that system signals can take, such as input-output signals, and is defined within the framework of the behavioral approach. Kaneko proposed a method for designing such controllers on data-driven control [2]. If the desired behavior is stable, matching the system's behavior with the desired behavior is equivalent to guaranteeing system stability. If a canonical controller can be designed using only data, stability in data-driven control can be guaranteed.

However, it is not always possible to design a canonical controller based on the available data. For example, designing a canonical controller is impossible if the system cannot achieve the desired behavior. Even when the system is theoretically capable of achieving the desired behavior, insufficient information in the data may prevent it from doing so. The first issue, the achievability of the desired behavior, is considered in Kaneko's study [2] based on the research by Julius et al. [3]. In contrast, the second issue, the requirements for data, still need to be clarified. Without precise data requirements, experiments for data acquisition must be conducted by trial and error. Therefore, it is essential to clarify the data requirements for the design of a canonical controller.

This study clarifies the data requirements for the design of a canonical controller for unknown systems. Clarifying these requirements can prevent unnecessary re-experiments for data acquisition, leading to both time and cost savings. It can also contribute to the broader discussion on the stability of data-driven control.

Since the discussion of canonical controllers is primarily conducted within the control theory framework known as the behavioral approach, Section II introduces the behavioral approach's fundamental concepts and the fundamental lemma related to data in the behavioral approach [4]. Section III explains canonical controllers and their design for known systems. Section IV clarifies the data requirements for designing of a canonical controller for unknown systems. Simulations are carried out to validate the clarified requirements.

II. Behavior Approach

This section explains the fundamental concepts and the fundamental lemma of the behavior approach in the context of data-driven control. The behavior approach is a control theory framework that defines a dynamical system as a set of time trajectories of system signals instead of conventional definitions as input-output relations.

A. Fundamental Concepts of the behavior Approach

In the behavior approach, a dynamical system Σ is defined by a triplet $\Sigma = (\mathbb{T}, \mathbb{W}, \mathcal{B})$; $\mathbb{T}$ is the set of times, where

[1,2,3]Yuki Ochiai, Triet Nguyen-Van and Shin Kawai are with Intelligent and Mechanical Interaction Systems, Graduate School of Science and Technology, University of Tsukuba, 1-1-1 Tennoudai, Tsukuba, Ibaraki, 305-8573 Japan {y.ochiai, kawai}@digicon-lab.esys.tsukuba.ac.jp, nguyen@g.iit.tsukuba.ac.jp

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$\mathbb{T} = \mathbb{R}$ for continuous-time systems and $\mathbb{T} = \mathbb{Z}$ for discrete-time systems; $\mathbb{W}$ is the signal space, which represents the set of values that the system signals can take at each time; $\mathcal{B} \subset \mathbb{W}^{\mathbb{T}}$ is the behavior, which represents the set of time trajectories of the system signals, where $\mathbb{W}^{\mathbb{T}}$ denotes the set of time functions from $\mathbb{T}$ to $\mathbb{W}$.

Consider $\mathcal{B}$ for a linear time-invariant system in continuous-time domain represented by the linear differential equation

$$R_0 w + R_1\left(\frac{dw}{dt}\right) + R_2\left(\frac{d^2w}{dt^2}\right) + \cdots + R_N\left(\frac{d^Nw}{dt^N}\right) = 0, \tag{1}$$

where w is a q-vector that summarizes the system signals, no input-output relationship between the elements is assumed, and $R_i \in \mathbb{R}^{p\times q}$. Using the polynomial matrix $R(\xi) = R_0 + R_1\xi + \cdots + R_N\xi^N$, (1) can be rewritten as

$$R\left(\frac{d}{dt}\right) w = 0. \tag{2}$$

The variable ξ in the polynomial matrix $R(\xi)$ corresponds to the differential operator d/dt. Then, $\mathcal{B}$ is the set of solutions to this differential equation as

$$\mathcal{B} = \left\{ w \in (\mathbb{R}^q)^{\mathbb{R}} \;\middle|\; R\left(\frac{d}{dt}\right) w = 0 \right\}. \tag{3}$$

In the behavior approach, dynamical systems are analyzed using this solution set. $R\left(\frac{d}{dt}\right) w = 0$ is referred to as the kernel representation of $\mathcal{B}$.

This paper treats the system Σ and the behavior $\mathcal{B}$ as equivalent and focuses on discrete-time systems where $w(k) \in (\mathbb{R}^q)^{\mathbb{Z}}$. In this case, ξ corresponds to the shift operator σ.

B. Fundamental Lemma

Given one long trajectory $w_d \in (\mathbb{R}^q)^{\mathbb{Z}}$ of a linear time-invariant system $\mathcal{B} \in \mathcal{L}^q$, multiple short trajectories can be generated by utilizing time invariance. The set $\mathcal{L}^q$ is the set of all linear time-invariant systems. In this paper, the subscript d denotes data, indicating that w_d represents data obtained from the system. Short trajectories $w|_L$ of length $L \in \{1, \ldots, T\}$ and the cut operator $|_L$ are defined as

$$w|_L := \begin{bmatrix} w(1)^\top & \cdots & w(L)^\top \end{bmatrix}^\top \in \mathbb{R}^{qL}. \tag{4}$$

By sequentially applying the shift operator σ and the cut operator $|_L$, $N = T - L + 1$ trajectories of length L is obtained as

$$(\sigma^0 w_d)\,|_L, (\sigma^1 w_d)\,|_L, \ldots, (\sigma^{T-L} w_d)\,|_L. \tag{5}$$

The $(qL) \times (T - L + 1)$ dimensional matrix formed by stacking these trajectories side by side is

$$\begin{aligned} \mathcal{H}_L(w_d) &:= \begin{bmatrix} (\sigma^0 w_d)\,|_L & (\sigma^1 w_d)\,|_L & \ldots & (\sigma^{T-L} w_d)\,|_L \end{bmatrix} \\ &= \begin{bmatrix} w_d(1) & w_d(2) & \cdots & w_d(T-L+1) \\ w_d(2) & w_d(3) & \cdots & w_d(T-L+2) \\ \vdots & \vdots & & \vdots \\ w_d(L) & w_d(L+1) & \cdots & w_d(T) \end{bmatrix}. \end{aligned} \tag{6}$$

This matrix is called the Hankel matrix of w_d with depth L. While $\mathcal{H}_L(w_d)$ can be defined for any $L \in \{1, \ldots, T\}$, it is required that $w_d(L)$ be included in at least the first row. Therefore, the number of columns must be greater than or equal to the number of rows, i.e., the following holds:

$$L \le L_{\max} := \left\lfloor \frac{T+1}{q+1} \right\rfloor. \tag{7}$$

Assume that w_d can be decomposed into m-input and p-output signals, and is defined as

$$\begin{aligned} w_d &:= (u_d, y_d) \\ &= \left(\begin{bmatrix} u_d(1) \\ y_d(1) \end{bmatrix}, \begin{bmatrix} u_d(2) \\ y_d(2) \end{bmatrix}, \cdots, \begin{bmatrix} u_d(T) \\ y_d(T) \end{bmatrix} \right) \in \mathcal{B}|_T, \end{aligned} \tag{8}$$

where $\mathcal{B}|_T$ is a set of trajectories of length T by applying the cut operator to the behavior. If $\mathcal{H}_L(u_d)$ is full row rank, i.e., $\mathrm{rank}\mathcal{H}_L(u_d) = mL$, the signal u_d is said to be persistently exciting of depth L. For u_d to be persistently exciting of depth L, it must have at least a sample length of $T_{\min} := (m+1)L - 1$.

The following result is known as the fundamental lemma and is used as a guideline for system identification and control input design.

Lemma 1 (Willems' Fundamental Lemma [4])**.** *For a linear time-invariant system $\mathcal{B} \in \mathcal{L}^q$ whose system signals can be decomposed as $w = (u, y)$ into input signal u and output signal y, let the following conditions hold:*

1) *$w_d = (u_d, y_d) \in \mathcal{B}|_T$ is a trajectory of $\mathcal{B}$.*
2) *The system $\mathcal{B}$ is controllable.*
3) *The input signal u_d of w_d is persistently exciting of depth $L + n(\mathcal{B})$, where $n(\mathcal{B})$ is the order of $\mathcal{B}$.*

Then, any trajectory w of $\mathcal{B}$ with L-sample length can be written as a linear combination of the columns of $\mathcal{H}_L(w_d)$, and any linear combination $\mathcal{H}_L(w_d)g$ is also a trajectory of $\mathcal{B}$, where $g \in \mathbb{R}^{T-L+1}$. Therefore,

$$\mathrm{image}\,\mathcal{H}_L(w_d) = \mathcal{B}|_L \tag{9}$$

holds.

III. Canonical Controller

In the behavior approach, the plant has two signals: controlled output and control input. Let w be the signal of the controlled output and c the signal of the control input, with their respective sets of possible values denoted as $\mathbb{W}^{\mathbb{T}}$ and $\mathbb{C}^{\mathbb{T}}$. The set of all possible plant behaviors is a subset of $\mathbb{W}^{\mathbb{T}} \times \mathbb{C}^{\mathbb{T}}$ and is called the full behavior $\mathcal{P}_f$ of the plant.

Figure 1 shows a conceptual diagram of the full behavior of the system.

The controller $\mathcal{C}$ is a subset of $\mathbb{C}^{\mathbb{T}}$, consisting of the signals c that the controller can take. The role of the controller is to restrict the controlled output w in the full behavior $\mathcal{P}_f$ using the control input c. Figure 2 shows a conceptual diagram showing the combination of the plant and the controller, where the controller restricts the full behavior of the system.

Consider one of the challenges in control theory: making the system match the desired behavior $\mathcal{S}$. If a controller $\mathcal{C}$ exists such that the full behavior $\mathcal{P}_f$ of the system equals the desired behavior $\mathcal{S}$, then $\mathcal{S}$ is said to be achievable by $\mathcal{C}$. One of the problems in controller design in the behavior approach is, given the full behavior $\mathcal{P}_f$ of the system and the desired behavior $\mathcal{S}$, to find a controller $\mathcal{C}$ that can achieve $\mathcal{S}$.

As a controller that solves this problem, the canonical controller was proposed by van der Schaft [5]. The canonical controller is constructed by combining the full behavior of the system with the desired behavior, as shown in Figure 3. By combining this with the plant, the set of control variables $\mathcal{C}$ is restricted so that the full behavior $\mathcal{P}_f$ of the system becomes equal to the desired behavior $\mathcal{S}$.

Let us consider one method for obtaining a canonical controller, restricting discussion to linear time-invariant systems in discrete-time domain. Let the full behavior of the system be $\mathcal{P}_f \subset (\mathbb{R}^{q+p})^{\mathbb{Z}}$ and $(w, c) \in \mathcal{P}_f$ be given by

$$R(\sigma)w = M(\sigma)c, \tag{10}$$

where $R \in \mathbb{R}^{\bullet \times q}[\xi]$ and $M \in \mathbb{R}^{\bullet \times p}[\xi]$. Let $\mathcal{S} \in (\mathbb{R}^q)^{\mathbb{Z}}$ be the desired behavior. In other words, the goal of the control is to find a $\mathcal{C}$ such that $\mathcal{P}_f$ and $\mathcal{S}$ become equal. If the existence of $K \in \mathbb{R}^{\bullet \times q}[\xi]$ such that $w \in \mathcal{S}$ is described by

$$K(\sigma)w = 0. \tag{11}$$

Then, the canonical controller is defined as the following set of trajectories:

$$\mathcal{C}_{can} = \left\{ c \in (\mathbb{R}^p)^{\mathbb{Z}} \mid \exists \tilde{w} \text{ s.t. } (\tilde{w}, c) \in \mathcal{P}_f, \tilde{w} \in \mathcal{S} \right\}. \tag{12}$$

The role of the canonical controller is to accurately realize the desired behavior $\mathcal{S}$ by connecting it to the plant.

Now, we consider whether the desired behavior $\mathcal{S}$ can be realized by connecting an appropriate controller to the full behavior $\mathcal{P}_f$ of the system. In other words, we consider whether a canonical controller can be designed. This problem is solved by the following theorem:

Theorem 1 (Controller Implementability Theorem [6])**.** *Given $\mathcal{P}_f \in \mathcal{L}^{q+p}$, the desired behavior $\mathcal{S}$ can be realized if and only if:*

$$\mathcal{N} \subset \mathcal{S} \subset \mathcal{P}. \tag{13}$$

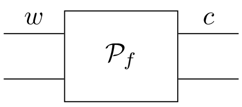

Fig. 1. A plant

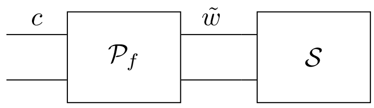

Fig. 2. A canonical controller

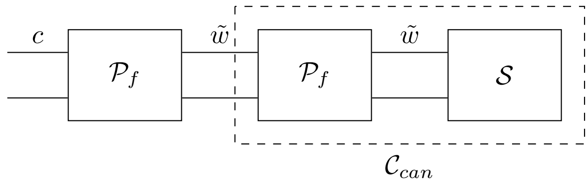

Fig. 3. Connection between a plant and a canonical controller

Here, $\mathcal{N} \in \mathcal{L}^w$ is defined as the hidden behavior:

$$\mathcal{N} := \left\{ w \in (\mathbb{R}^q)^{\mathbb{Z}} \mid (w, 0) \in \mathcal{P}_f \right\}. \tag{14}$$

Additionally, $\mathcal{P} \in \mathcal{L}^w$ is defined as the manifest plant behavior:

$$\mathcal{P} := \left\{ w \in (\mathbb{R}^q)^{\mathbb{Z}} \mid \exists c \text{ such that } (w, c) \in \mathcal{P}_f \right\}. \tag{15}$$

At this point, the canonical controller $\mathcal{C}_{can}$ can be parameterized using the matrices R and M. First, find a polynomial matrix Q such that:

$$QR = K. \tag{16}$$

Substituting equation (16) into equations (10) and (11), we obtain:

$$QMc = 0. \tag{17}$$

From this equation, the kernel representation of the canonical controller can be derived.

IV. Main Results

This section clarifies the requirements under which a canonical controller can be designed using driving data. The previous section explained the design of canonical controllers for known systems based on discussions from prior research. Kaneko's prior research also proposed methods for designing canonical controllers for unknown systems using finite-length driving data. However, these methods do not specify the requirements for the driving data for the design of a canonical controller. As mentioned in the introduction, when the conditions for the driving data are unclear, experiments to obtain them incur unnecessary costs. Therefore, a quantitative criterion for the required driving data is needed.

The problem to be considered in this section is as follows.

Problem Statement. *Clarify the requirements for the driving data w_d that satisfy the following equation:*

$$\text{image}\, \mathcal{H}_L(w_d) = \mathcal{S}|_L, \tag{18}$$

where $\mathcal{S}|_L$ is the desired behavior $\mathcal{S}$ of length L, and the following data and behavior are given:

- *Driving data $w_d := (u_d, y_d)$ of the unknown plant*

- *Desired behavior $\mathcal{S}$*

Based on Willems' fundamental lemma, lemma 1, this problem can be solved as follows:

Theorem 2. *Let the desired behavior $\mathcal{S}$ and the driving data $w_d = (u_d, y_d)$ of a linear time-invariant system $\mathcal{B}$ be given. It should be noted that the mathematical model of the linear time-invariant system $\mathcal{B}$ is unknown, and only the driving data w_d is available. If the following conditions hold:*

1) *The desired behavior $\mathcal{S}$ is linear and time-invariant.*
2) *$w_d = (u_d, y_d) \in \mathcal{B}|_T$ is a trajectory of $\mathcal{S}$.*
3) *The system $\mathcal{S}$ is controllable.*
4) *The input component u_d of w_d is persistently exciting of depth $L + n(\mathcal{S})$.*

Then, (18) holds, i.e., it is possible to design a canonical controller based on the driving data w_d of the unknown plant. Simultaneously, any L-sample length trajectory w of $\mathcal{S}$ can be written as a linear combination of the columns of $\mathcal{H}_L(w_d)$, and any linear combination $\mathcal{H}_L(w_d)g$ is also a trajectory of $\mathcal{S}$.

Proof. Willems' fundamental lemma discusses known linear time-invariant systems and their driving data, and it has been proven in [4]. Theorem 2 derived in this paper deals with the desired behavior $\mathcal{S}$ and the driving data of an unknown plant $\mathcal{B}$. Theorem 2 replaces the known linear time-invariant system in Willems' fundamental lemma with the desired behavior $\mathcal{S}$. This replacement can be considered as a restriction of the scope of Willems' fundamental lemma, considering that the condition (13) must be satisfied for achievability. Therefore, Theorem 2 is proven in the same way as Willems' fundamental lemma in [4]. □

Remark 1. *Since the control objective is to realize the desired behavior $\mathcal{S}$ for a finite period in practical applications, it is essential to set the data length L to the target finite period. While the length of the driving data w_d depends on the characteristics of the desired behavior $\mathcal{S}|_L$, it must be at least $(m+1)L-1$.*

Remark 2. *Theorem 2 is a sufficient condition for designing a canonical controller; it may be possible to design a canonical controller without the conditions of Theorem 2. For instance, if the behavior converges at some point, it is sufficient to achieve the behavior up to that point, after which the remaining behavior can also be achieved.*

V. Simulation

While the design of a canonical controller is feasible under Theorem 2, it imposes conservative conditions as it provides a sufficient condition. Practical considerations, therefore, should also be addressed. To satisfy the conditions of Theorem 2, the relationship between the unknown plant $\mathcal{B}$ and the desired behavior $\mathcal{S}$ must be such that $\mathcal{S} \subset \mathcal{B}$. It is challenging to always define an appropriate desired behavior $\mathcal{S}$ for an unknown plant $\mathcal{B}$. Hence, Theorem 2 can be regarded as a theory better suited for analysis than for design.

Suppose that a controller $\mathcal{C}$ has been designed for an unknown plant $\mathcal{B}$ using a data-driven control approach to approximate the response of a predefined reference model $\mathcal{D}$. In practice, it is challenging to make the response of the plant $\mathcal{B}$ fully match that of the reference model $\mathcal{D}$. Therefore, instead of using the predefined reference model $\mathcal{D}$, the open-loop system $\mathcal{P}$ consisting of the unknown plant $\mathcal{B}$ and the controller $\mathcal{C}$ is assumed to be the desired behavior $\mathcal{S}$. Under this assumption, the controller $\mathcal{C}$ can be regarded as a canonical controller $\mathcal{C}_{\text{can}}$ that achieves the desired behavior $\mathcal{S}$ for the unknown plant $\mathcal{B}$.

It should be noted that, since the information available to us is limited to finite measured data, the performance of the canonical controller can only be analyzed within a finite interval L that satisfies the conditions of Theorem 2. Therefore, by analyzing the finite-length behavior $\mathcal{P}_L$ of the open-loop system, it becomes possible to evaluate how the controller $\mathcal{C}$ achieves a specific behavior for the unknown plant $\mathcal{B}$. In the following, simulations using Matlab/Simulink are conducted to verify the conditions under which the controller $\mathcal{C}$, designed for an unknown plant $\mathcal{B}$, can be regarded as a canonical controller.

A. Simulation Setup

The plant is given by

$$G(s) = \frac{12s + 8}{20s^4 + 113s^3 + 147s^2 + 62s + 8}, \tag{19}$$

and a zero-order hold is used for the D/A converter. In practice, the plant is assumed to be unknown, and only the data w_d obtained from the plant is available for design. The data w_d consists of input and output signals, u_d and y_d, representing the control input and the controlled output, respectively. This data w_d is obtained from a feedback system consisting of the plant and the initial controller $C(\rho_0)$ with a sampling period of $T_s = 0.01$.

The controller $C(\rho)$ is a discrete-time PID controller given by

$$C(\rho) = \rho_1 + \rho_2 \frac{T_s}{z-1} + \rho_3 \frac{z-1}{\rho_4 + \frac{T_s z}{z-1}}. \tag{20}$$

The derivative term is an approximate differentiator and ρ_4 functions as a frequency filter. The controller has four parameters; the initial values are set as $\rho = \begin{bmatrix} 3 & 0.5 & 0 & 0 \end{bmatrix}$.

The following transfer function

$$T_{des} = \frac{1}{(0.5s+1)^2} \tag{21}$$

with a zero-order hold produces the desired response, and the step response y_{des} is used for the simulation.

The simulation time is set to 15 seconds, and both w_d and y_{des} have a length of 1,500. Figure 4 shows the desired response y_{des} and the response y_d with the initial controller.

B. Results and Discussion

Parameter tuning aimed to minimize the evaluation function:

$$J(\rho) = \|y_{des} - y(t, \rho)\|^2 . \tag{22}$$

The tuning continued until the evaluation function stabilized. When the evaluation function reached a value of 0.00257115, the following parameters were obtained:

$$\rho = \begin{bmatrix} 6.0732 & 0.9817 & 7.0342 & 0.0471 \end{bmatrix} . \tag{23}$$

The simulation results obtained by applying these parameters to the controller are shown in Fig. 5. It can be observed that the system response y is sufficiently close to the desired response y_{des}. However, some error remains, making it impossible to consider the controller a canonical controller with respect to the reference model. Therefore, as mentioned earlier, the open-loop system consisting of the unknown plant and the designed controller is assumed to be the desired behavior. Under this assumption, the controller can be considered a canonical controller within the finite interval.

Since the finite interval is determined according to the conditions of Theorem 2, it is sufficient to compute the maximum L for which the control input u applied to the plant satisfies the persistency of excitation condition. Figure 6 shows the plot of the control input u, where $L = 105$ is the maximum value for which u satisfies the persistency of excitation condition. This indicates that the designed controller is a canonical controller that enables the unknown plant to achieve all finite-length behaviors $\mathcal{S}|_L$ of the desired behavior $\mathcal{S}$ for $L = 105$.

Given that the sampling period is 0.01, this corresponds to only 1.05 seconds of matching behavior. Considering the rise time of step signal is 1 second, the value of the finite length L is extremely small. To address this, random disturbances were added to the step input to increase the value of the

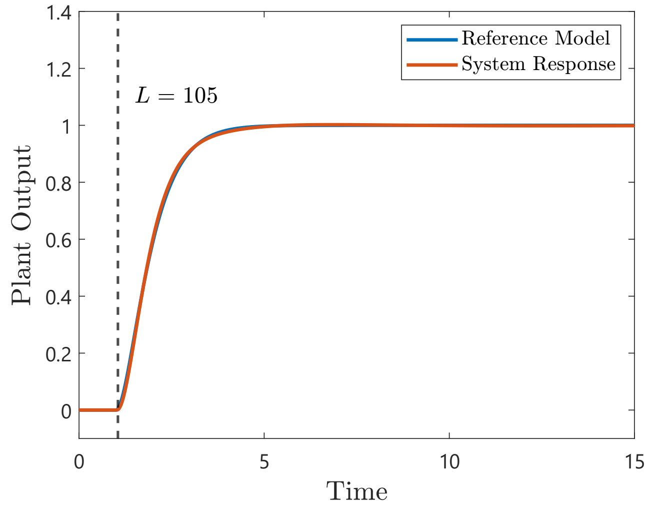

Fig. 5. Comparison of Desired Response and System Response

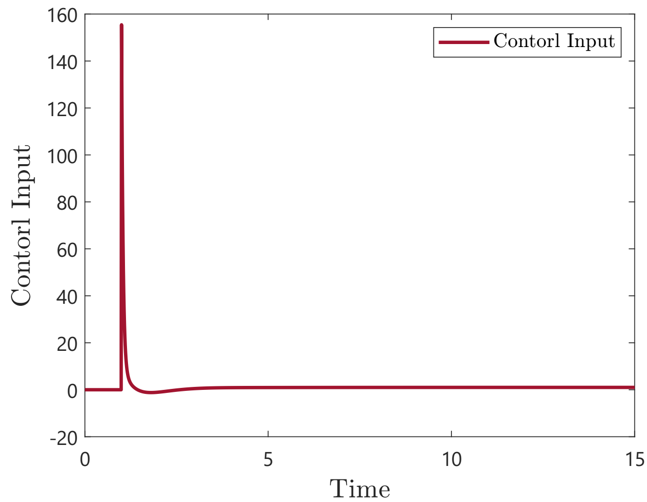

Fig. 6. Control Input

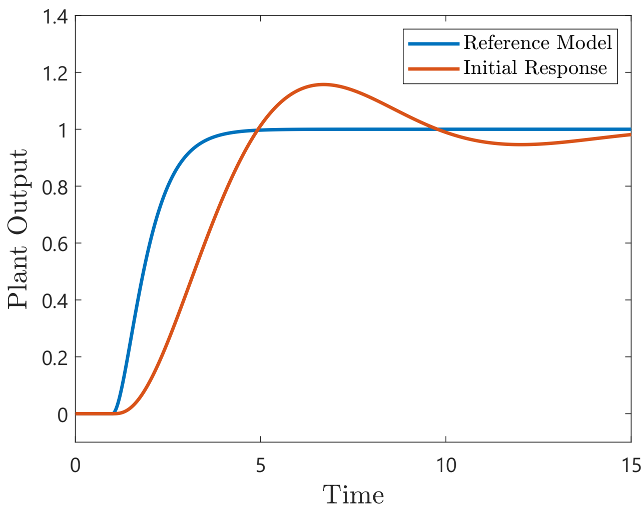

Fig. 4. Comparison of Desired Response and System Response Under Initial Conditions

finite length L. The results are shown in Fig. 7. As in Fig. 5, it can be observed that the system response closely matches the desired response y_{des}. Similarly, the open-loop system consisting of the unknown plant and the designed controller is regarded as the desired behavior. Figure 8 shows the control input applied to the system, where $L = 750$ is the maximum value for which the input satisfies the persistency of excitation condition. Although the controller and the plant remained unchanged, the finite length over which it can be regarded as a canonical controller was successfully increased.

By analyzing the desired behavior, it becomes possible to evaluate the characteristics of the response achieved by the canonical controller for the unknown plant. Thus, an increase in the finite length corresponds to an expansion of the analyzable finite interval. For example, by employing the approach proposed in [7] to compute the $L2$ gain, it becomes possible to discuss the stability of controllers designed using data-driven control methods within a finite interval.

The following presents future challenges to improve prac-

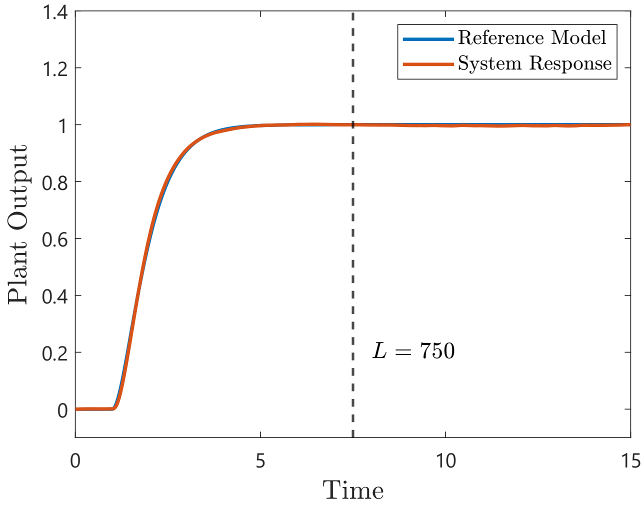

Fig. 7. Comparison of Desired Response and System Response (With Disturbance)

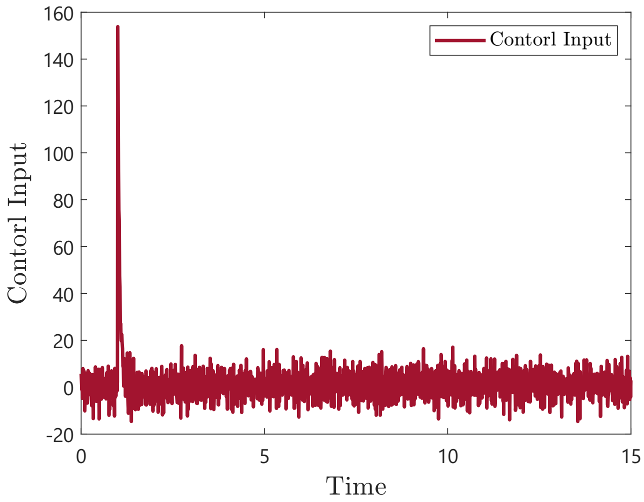

Fig. 8. Control Input (With Disturbance)

ticality. The data length required for the canonical controller to match the desired response is a key consideration. First, if the response converges to a steady state midway, it is unnecessary to match all data after convergence. Moreover, if the transient response can be represented by periodic behavior or a superposition of such behaviors, the required data length can be further reduced. By doing so, the conditions for the depth at which u satisfies the persistency of excitation can be relaxed, leading to increased practicality. Therefore, theoretical investigations are needed to reduce the data length required to match the desired response.

VI. Conclusion

This study has clarified the data requirements for designing a canonical controller for unknown systems based on Willems' fundamental lemma. Furthermore, simulations examined the practical applicability of the clarified data requirements. The simulation results revealed that, while the data requirements presented in Theorem 2 are concrete, they impose conservative conditions, which present certain constraints in practical design.

The simulation results also demonstrated that adding disturbances to the reference input can extend the finite interval that satisfies the design requirements for a canonical controller. Extending this finite interval corresponds to broadening the range in which the impact of the canonical controller on an unknown system can be analyzed. In future work, two key challenges are identified to enhance the practicality of the clarified requirements. The first is analyzing the desired behavior achieved using the canonical controller for an unknown system. The second is reducing the data length required to achieve the desired response, which could further improve the practicality of the design requirements.

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References

[1] C. De Persis and P. Tesi, "Formulas for data-driven control: Stabilization, optimality, and robustness," *IEEE Transactions on Automatic Control*, vol. 65, no. 3, pp. 909–924, 2020.

[2] O. Kaneko, "On linear canonical controllers within the unfalsified control framework," *IFAC Proceedings Volumes*, vol. 41, no. 2, pp. 12 279–12 284, 2008.

[3] A. Julius, J. Willems, M. Belur, and H. Trentelman, "The canonical controllers and regular interconnection," *Systems & Control Letters*, vol. 54, no. 8, pp. 787–797, 2005.

[4] J. C. Willems, P. Rapisarda, I. Markovsky, and B. L. De Moor, "A note on persistency of excitation," *Systems & Control Letters*, vol. 54, no. 4, pp. 325–329, 2005.

[5] A. van der Schaft, "Achievable behavior of general systems," *Systems & Control Letters*, vol. 49, no. 2, pp. 141–149, 2003.

[6] J. Willems and H. Trentelman, "Synthesis of dissipative systems using quadratic differential forms: Part i," *IEEE Transactions on Automatic Control*, vol. 47, pp. 53–69, 2002.

[7] A. Romer, J. Berberich, J. Kohler, and F. Allgower, "One-Shot Verification of Dissipativity Properties From Input–Output Data," *IEEE Control Systems Letters*, vol. 3, no. 3, pp. 709–714, 2019.