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Cooperative Wind Disturbance Estimation by Multiple Drones in the Presence of Torque Disturbances

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Abstract— **This paper deals with cooperative wind disturbance estimation by multiple quadrotor drones. The authors have developed a cooperative wind disturbance estimation method combining the optimal control and adaptive control. However, only disturbances affecting translational motion have been considered and torque disturbances influencing rotational motion have not been taken into account. This paper addresses a cooperative disturbance estimation problem in a more realistic situation where torque disturbances exist as well. The two types of disturbances are estimated and compensated simultaneously. A disturbance estimator is designed based on the Lyapunov stability theory. Numerical simulations show the effectiveness of the proposed method and also indicate the necessity of simultaneous estimation of translational disturbances and torque disturbances.**

I. INTRODUCTION

Multirotor drones are employed in a wide range of applications such as transportation and surveillance. Furthermore, their role is expected to expand more and more in the future. The use of multiple drones offers redundancy. This not only expands the range of achievable missions, but also enhances efficiency by enabling the completion of more tasks in less time. Moreover, it improves reliability because, if one drone leaves, the other drones will still be functional [1], [2], [3].

Drones are also expected to play a vital role in wind measurements particularly for acquiring wind information in the atmospheric boundary layer (ABL). The ABL is the lower part of the convective layer. This layer is influenced by heat and friction from the Earth's surface, which leads to environmental changes. To understand these mechanisms, wind information is necessary in meteorological research [4], [5]. Traditional tools for acquiring wind information are meteorological towers, weather balloons, manned aircraft, and so on. Compared to these tools, drones have advantages of limited space on the ground, low-cost operation, and ease of observation while moving. Therefore, drones can be regarded as an effective means for measuring wind. Wind information is also beneficial for operation of drones in severe weather conditions. If wind disturbances are estimated, compensating for their effects by suitable motion control improves the reliability of flight of drones [6], [7].

Several studies have addressed estimation of wind disturbances using a single drone. These studies include those using additional sensors [8], [9] and those exploiting drone's motion information only [10], [11], [12], [13]. Needless to say, omitting extra sensors for wind measurement contributes to increase of payload of a drone. Our research group aims at acquiring wind information from motion of multiple drones. The use of multiple drones enables us to obtain relative information among drones. The authors have developed a distributed cooperative wind disturbance estimation method for translational motion by combining the optimal control and adaptive control [14]. However, it does not account for torque disturbances affecting rotational motion. If torque disturbances are also cooperatively estimated and compensated by suitable motion control, wind measurement using drones will be carried out more accurately.

For this reason, a cooperative torque estimation method is developed in this paper. To achieve this objective, we employ a control law for rotational motion designed based on the special orthogonal group $\mathsf{SO}(3)$ [15]. An update law for torque disturbance estimation is formulated by means of the Lyapunov stability theory. As a result, we can simultaneously estimate and cancel two types of disturbances: disturbances affecting translational motion and torque disturbances influencing rotational motion. Numerical simulations support the effectiveness of the proposed approach.

This paper is organized as follows. In Section II, we introduce a dynamic model of drones under disturbances and formulate the problem to be solved in this paper. Section III reviews our previous result for disturbance estimation in translational motion. For rotational motion, a control law and an update law for torque disturbance estimation are designed in Section IV. In Section V, we demonstrate the effectiveness of the proposed approach through numerical simulations. A comparison of results for cases with and without torque disturbance estimation is also provided.

Notation: Let $I_n \in \mathbb{R}^{n\times n}$ be the identify matrix of order n. The n-dimensional vector of all ones is denoted by $\mathbf{1}_n = (1,1,\dots,1)^\top \in \mathbb{R}^n$. The symbol $\otimes$ represents the Kronecker product. The hat map $(\cdot)^\times : \mathbb{R}^3 \to \mathbb{R}^{3\times 3}$ is defined by

$$a^\times = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}, \qquad (1)$$

for a three-dimensional vector $a = (a_1, a_2, a_3)^\top \in \mathbb{R}^3$. The vee map $(\cdot)^\vee : \mathbb{R}^{3\times 3} \to \mathbb{R}^3$ is defined by

$$\begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}^\vee = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}. \qquad (2)$$

It is clear that, for any $a \in \mathbb{R}^3$, we have $(a^\times)^\vee = a$.

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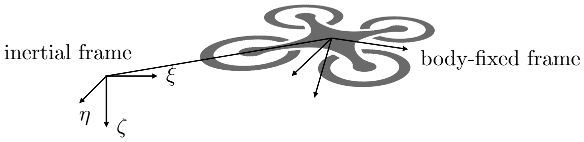

Fig. 1. The earth-fixed inertial frame and the body-fixed frame.

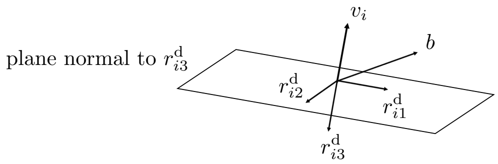

Fig. 2. Virtual input and orthogonal normalized vectors

II. PROBLEM SETTING

Let n be a natural number greater than or equal to 2. Assume that there are n quadrotor drones. We choose an earth-fixed inertial frame and a body-fixed frame to represent the dynamics of each drone. The inertial frame is common to all drones. The origin of each body-fixed frame is located at the center of gravity of each drone as shown in Fig. 1. Quantities of each drone are specified with the subscript i and, unless otherwise stated, i takes every value in $\{1,2,\ldots,n\}$. The center of gravity of the ith drone expressed in the inertial frame is denoted by $p_i(t) = (\xi_i, \eta_i, \zeta_i)^\top$. To represent the attitude, the special orthogonal group $\mathsf{SO}(3) = \{R \in \mathbb{R}^{3\times 3} | R^\top R = I, \det R = 1\}$ is used. The attitude of the ith drone is represented as $R_i(t) \in \mathsf{SO}(3)$, which is the rotation matrix from the inertial frame to the body-fixed frame. The angular velocity vector of the ith drone expressed in the body-fixed frame is denoted by $\omega_i(t) \in \mathbb{R}^3$. The equation of motion for the ith drone under aerodynamic disturbances is given by

$$m_i \ddot{p}_i = m_i g e_3 - T_i R_i^\top e_3 + d_i, \tag{3}$$

$$\dot{R}_i = -\omega_i^\times R_i, \tag{4}$$

$$J_i \dot{\omega}_i = -\omega_i \times (J_i \omega_i) + \tau_i + d_{\mathrm{t}i}, \tag{5}$$

where $e_3 := (0,0,1)^\top$, $g > 0$ is the gravitational acceleration, and $m_i > 0$ and $J_i = J_i^\top \in \mathbb{R}^{3\times 3}$ are the mass and the moment of inertia of the ith drone, respectively. The magnitude of the total thrust and the torque input applied to the ith drone are denoted by $T_i(t) \geq 0$ and $\tau_i(t) \in \mathbb{R}^3$, respectively.

The translational motion is affected by an unknown disturbance $d_i(t) \in \mathbb{R}^3$. Similarly, the rotational motion is influenced by an unknown torque disturbance $d_{\mathrm{t}i}(t) \in \mathbb{R}^3$. It is assumed that d_i and $d_{\mathrm{t}i}$ consist of steady terms $\bar{w}_i \in \mathbb{R}^3$, $\bar{w}_{\mathrm{t}i} \in \mathbb{R}^3$ and unsteady terms $w_i(t) \in \mathbb{R}^3$, $w_{\mathrm{t}i}(t) \in \mathbb{R}^3$, namely,

$$d_i(t) = \bar{w}_i + w_i(t), \tag{6}$$

$$d_{\mathrm{t}i}(t) = \bar{w}_{\mathrm{t}i} + w_{\mathrm{t}i}(t). \tag{7}$$

We also assume that there exist $w_0 > 0$ and $w_{\mathrm{t}0} > 0$ such that $\|w_i(t)\| < w_0$ and $\|w_{\mathrm{t}i}(t)\| < w_{\mathrm{t}0}$ for all $t \geq 0$.

For cooperative disturbance estimation by multiple drones, we address the formation control problem. To handle this problem, a new variable $\tilde{p}_i(t)$ is defined as

$$\tilde{p}_i(t) = p_i(t) - \begin{bmatrix} l_{\xi_i} \\ l_{\eta_i} \\ l_{\zeta_i} \end{bmatrix} = \begin{bmatrix} \xi_i(t) - l_{\xi_i} \\ \eta_i(t) - l_{\eta_i} \\ \zeta_i(t) - l_{\zeta_i} \end{bmatrix}, \tag{8}$$

where $l_{\xi_i}, l_{\eta_i}, l_{\zeta_i} \in \mathbb{R}$ are design parameters that determine the shape of the formation. Differentiating (8) twice with respect to t yields $\ddot{\tilde{p}}_i = \ddot{p}_i$. Then, it follows from (3) that

$$m_i \ddot{\tilde{p}}_i = m_i g e_3 - T_i R_i^\top e_3 + d_i. \tag{9}$$

Since l_{ξ_i}, l_{η_i}, l_{ζ_i} are constants, the shift (8) does not change the equation of translational motion.

The control objective for translational motion is divided into two parts. One is to design a control law that achieves

$$\lim_{t\to\infty} |\dot{\tilde{p}}_i(t)| = 0, \quad \lim_{t\to\infty} |\tilde{p}_i(t) - \tilde{p}_j(t)| = 0, \tag{10}$$

for any $i, j \in \{1,2,\cdots,n\}$. However, since the disturbance includes an unsteady term, it is impossible to achieve (10) completely. Therefore, we attempt to make $|\dot{\tilde{p}}_i(t)|$ and $|\tilde{p}_i(t) - \tilde{p}_j(t)|$ converge to a neighborhood of 0. The other objective is disturbance estimation. This aims at estimating the steady term $\bar{w}_i$. The disturbance estimation error $\tilde{d}_i(t) \in \mathbb{R}^3$ is defined as

$$\tilde{d}_i(t) = \bar{w}_i - \hat{d}_i(t) \in \mathbb{R}^3, \tag{11}$$

where $\hat{d}_i$ is an estimate of the steady component $\bar{w}_i$. An estimation law is designed so that

$$\lim_{t\to\infty} \tilde{d}_i(t) = 0, \tag{12}$$

for any $i \in \{1,2,\cdots,n\}$.

To design a cooperative control law that achieves the translational control objectives, the term $-(1/m_i)T_i R_i^\top e_3$ on the right-hand side of (9) is regarded as the virtual control input $v_i(t) = (v_{i1}(t), v_{i2}(t), v_{i3}(t))^\top \in \mathbb{R}^3$. Once a control law for the virtual control input v_i is obtained, the reference thrust $T_i^{\mathrm{d}}(t) \in \mathbb{R}$ and reference attitude $R_i^{\mathrm{d}}(t) \in \mathsf{SO}(3)$ are determined so that the following relation holds:

$$-\frac{1}{m_i} T_i^{\mathrm{d}} R_i^{\mathrm{d}\top} e_3 = v_i. \tag{13}$$

We follow the approach in [15]. It is clear from (13) that the reference thrust is given by

$$T_i^{\mathrm{d}} = m_i \sqrt{v_{i1}^2 + v_{i2}^2 + v_{i3}^2}. \tag{14}$$

We next specify the reference rotation matrix R_i^{d}. Let $r_{i1}^{\mathrm{d}}(t)$, $r_{i2}^{\mathrm{d}}(t)$, $r_{i3}^{\mathrm{d}}(t) \in \mathbb{R}^3$ be orthonormal vectors depicted in Fig 2. These are given as

$$r_{i3}^{\mathrm{d}} = -\frac{v_i}{\|v_i\|}, \quad r_{i2}^{\mathrm{d}} = \frac{r_{i3}^{\mathrm{d}} \times b}{\|r_{i3}^{\mathrm{d}} \times b\|}, \quad r_{i1}^{\mathrm{d}} = r_{i2}^{\mathrm{d}} \times r_{i3}^{\mathrm{d}}, \tag{15}$$

where the desired direction $b(t) \in \mathbb{R}^3$, which determines the orientation of the first and second axes of the body-fixed frame, is not parallel to r_{i3}^{d}. The reference attitude is then determined as

$$R_i^{\mathrm{d}} = \begin{bmatrix} r_{i1}^{\mathrm{d}} & r_{i2}^{\mathrm{d}} & r_{i3}^{\mathrm{d}} \end{bmatrix}^\top. \tag{16}$$

To evaluate the difference between R_i and R_i^{d}, a non-negative-valued error function of attitude is defined as

$$\Psi_i(R_i(t), R_i^{\mathrm{d}}(t)) = \frac{1}{2}\operatorname{tr}\left[I - R_i^{\mathrm{d}}(t)R_i(t)^\top\right] \geq 0. \tag{17}$$

Then, the control objective for attitude motion is to design a control law that achieves

$$\lim_{t\to\infty} \Psi_i(R_i(t), R_i^{\mathrm{d}}(t)) = 0. \tag{18}$$

As in the case of translational motion, we attempt to realize that R_i converges in the neighborhood of R_i^{d}. The torque disturbance estimation error is defined as

$$\tilde{d}_{\mathrm{t}i}(t) = \bar{w}_{\mathrm{t}i} - \hat{d}_{\mathrm{t}i}(t) \in \mathbb{R}^3, \tag{19}$$

where $\hat{d}_{\mathrm{t}i}$ is an estimate of the steady component $\bar{w}_{\mathrm{t}i}$. The estimation update law is designed to achieve

$$\lim_{t\to\infty} \tilde{d}_{\mathrm{t}i}(t) = 0 \tag{20}$$

for any $i \in \{1, 2, \cdots, n\}$.

III. CONTROL OF TRANSLATIONAL MOTION

In this section, we briefly review our previous result [14]. The details are presented in [14]. The control law is designed by means of the hierarchical optimal control [16]. The update law, which estimates the steady term of the disturbance, is formulated based on the Lyapunov stability theory.

A. Translational Control Law

As described in the previous section, the translational motion is controlled by the virtual input. The state space representation of (9) with the virtual input v_i is given by

$$\begin{aligned}\dot{x}_i &= Ax_i + B\left(v_i + ge_3 + \frac{1}{m_i}d_i\right) \\ &= Ax_i + B\left(v_i + ge_3 + \frac{1}{m_i}\hat{d}_i\right) + B\left(\frac{1}{m_i}(\tilde{d}_i + w_i)\right).\end{aligned} \tag{21}$$

where $x_i = (\tilde{p}_i, \dot{\tilde{p}}_i)^\top$ for each $i \in \{1, 2, \cdots, n\}$. The matrices A and B are given by

$$A = \begin{bmatrix} 0 & I_3 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ I_3 \end{bmatrix}.$$

We introduce a new variable u_i as

$$u_i = v_i + ge_3 + \frac{1}{m_i}\hat{d}_i. \tag{22}$$

All the subsystems are represented as follows:

$$\dot{x} = (I_n \otimes A)x + (I_n \otimes B)u + (M^{-1} \otimes B)(\tilde{d} + w), \tag{23}$$

where the overall state x, input u, estimation error $\tilde{d}$, unsteady disturbance w and mass M are defined by

$$\begin{aligned} &x = \left(x_1^\top, x_2^\top, \ldots, x_n^\top\right)^\top, \quad u = \left(u_1^\top, u_2^\top, \ldots, u_n^\top\right)^\top, \\ &\tilde{d} = \left(\tilde{d}_1^\top, \tilde{d}_2^\top, \ldots, \tilde{d}_n^\top\right)^\top, \quad w = \left(w_1^\top, w_2^\top, \ldots, w_n^\top\right)^\top, \\ &M = \operatorname{diag}(m_1, m_2, \ldots, m_n). \end{aligned}$$

If unknown terms of the disturbance are excluded, the state space representation (21) becomes

$$\dot{x} = (I_n \otimes A)x + (I_n \otimes B)u. \tag{24}$$

To design a control law that achieves the translational control objectives, the performance output z is set as

$$z = \begin{bmatrix} \dot{\tilde{p}}_1 \\ \dot{\tilde{p}}_2 \\ \vdots \\ \dot{\tilde{p}}_n \\ \tilde{p}_1 - \tilde{p}_2 \\ \vdots \\ \tilde{p}_1 - \tilde{p}_n \end{bmatrix} = Cx, \tag{25}$$

where the matrix $C \in \mathbb{R}^{(6n-3)\times 6n}$ is defined by

$$C = \begin{bmatrix} I_n \otimes \begin{bmatrix} 0 & I_3 \end{bmatrix} \\ \begin{bmatrix} \mathbf{1}_{n-1} & -I_{n-1} \end{bmatrix} \otimes \begin{bmatrix} I_3 & 0 \end{bmatrix} \end{bmatrix}. \tag{26}$$

The first n block components of z correspond to the first control objective in (10). The last $(n-1)$ block components of z are to achieve the second objective in (10). The ideal translational control law is determined so that the following quadratic cost function is minimized:

$$J = \int_0^\infty \left(z(t)^\top Qz(t) + u(t)^\top Ru(t)\right)dt. \tag{27}$$

The output and the input weights are given by

$$Q = \begin{bmatrix} I_n \otimes Q_{\mathrm{id}} & 0 \\ 0 & I_{n-1} \otimes Q_{\mathrm{cp}} \end{bmatrix}, \quad R = I_n \otimes R_{\mathrm{id}}, \tag{28}$$

where $Q_{\mathrm{id}} = Q_{\mathrm{id}}^\top \in \mathbb{R}^{3\times 3}, Q_{\mathrm{cp}} = Q_{\mathrm{cp}}^\top \in \mathbb{R}^{3\times 3}, R_{\mathrm{id}} = R_{\mathrm{id}}^\top \in \mathbb{R}^{3\times 3}$ are positive-definite matrices. Since it is a standard linear quadratic (LQ) optimal control problem, the optimal control input is easily obtained as

$$u = -R^{-1}(I_n \otimes B^\top)Px, \tag{29}$$

where $P \in \mathbb{R}^{6n\times 6n}$ is the positive-semidefinite solution to the Riccati equation

$$\begin{aligned} &P(I_n \otimes A) + (I_n \otimes A)^\top P \\ &\qquad - P\left(I_n \otimes (BR_{\mathrm{id}}^{-1}B^\top)\right)P + C^\top QC = 0. \end{aligned} \tag{30}$$

According to [14], there exist $P_1, P_2, P_3 \in \mathbb{R}^{6\times 6}$ such that

$$P = I_n \otimes P_1 + L_{\mathrm{s}} \otimes P_2 + L_{\mathrm{s}} \otimes P_3, \tag{31}$$

where L_{s} and L_{c} are defined by

$$L_{\mathrm{s}} = \begin{bmatrix} (n-1) & -\mathbf{1}_{n-1}^\top \\ -\mathbf{1}_{n-1} & I_{n-1} \end{bmatrix}, \quad L_{\mathrm{c}} = nI_n - \mathbf{1}_n\mathbf{1}_n^\top. \tag{32}$$

Then, the optimal control law (29) becomes

$$u_1 = -R_{\mathrm{id}}^{-1}B^\top P_1 x_1 - nR_{\mathrm{id}}^{-1}B^\top(P_2 + P_3)(x_1 - x_{\mathrm{ave}}), \tag{33}$$

$$\begin{aligned} u_i = &-R_{\mathrm{id}}^{-1}B^\top P_1 x_i - R_{\mathrm{id}}^{-1}B^\top P_2(x_i - x_1) \\ &- nR_{\mathrm{id}}^{-1}B^\top P_3(x_i - x_{\mathrm{ave}}), \qquad i \in \{2, 3, \ldots, n\}, \end{aligned} \tag{34}$$

where x_{ave} is the average state defined by

$$x_{\mathrm{ave}} = \frac{1}{n}\sum_{i=1}^n x_i. \tag{35}$$

The control laws (33) and (34) are the ideal translational cooperative control law that achieves the translational control objective (10).

B. Cooperative disturbance estimation

We next design an update law for the disturbance estimation based on Lyapunov stability theory. From (23) and (29), the closed-loop system, including unknown disturbance, is given by

$$\dot{x} = \Big((I_n \otimes A) - \big(I_n \otimes (BR_{\mathrm{id}}^{-1}B^\top)\big)P\Big)x + (M^{-1} \otimes B)(\tilde{d} + w). \tag{36}$$

If estimation is accurate enough so that $\tilde{d}$ takes small values and the unsteady term w is small enough, then the control (10) is achieved. Hence, a strategy to design an update law is to update the estimate of the steady terms of the disturbance so that the closed-loop system has a stable behavior.

Let a candidate of the Lyapunov function be

$$V(x) = x^\top P x + \tilde{d}^\top \Gamma^{-1} \tilde{d}. \tag{37}$$

where $\Gamma = \Gamma^\top \in \mathbb{R}^{3n\times 3n}$ is a design parameter for disturbance estimation law. In this paper, Γ is set as

$$\Gamma = \text{blk-diag}\,(\Gamma_1, \Gamma_2, \dots, \Gamma_n) \tag{38}$$

for some positive definite matrices $\Gamma_i = \Gamma_i^\top \in \mathbb{R}^{3\times 3}$, $i \in \{1, 2, \dots, n\}$. From (11), (30) and (36), the time derivative of V is given by

$$\begin{aligned}\dot{V}(x) &= \dot{x}^\top P x + x^\top P \dot{x} + 2\tilde{d}^\top \Gamma^{-1} \dot{\tilde{d}} \\ &= -x^\top C^\top Q C x - x^\top P\big(I_n \otimes (BR_{\mathrm{id}}^{-1}B^\top)\big)Px \\ &\quad + 2w^\top (M^{-1} \otimes B^\top)Px \\ &\quad + 2\tilde{d}^\top \Big((M^{-1} \otimes B^\top)Px - \Gamma^{-1}\dot{\hat{d}}\Big),\end{aligned} \tag{39}$$

where $\hat{d} = \big(\hat{d}_1^\top, \hat{d}_2^\top, \cdots, \hat{d}_n^\top\big)^\top$. The first and second terms are negative definite. However, the remaining terms is indefinite. The third term remains because it includes an unknown value w, but we can eliminate the fourth term. Hence, the update law for the disturbance estimation in translational motion is formulated as

$$\dot{\hat{d}} = \Gamma(M^{-1} \otimes B^\top)Px. \tag{40}$$

Substituting (31) into the update law (40) gives

$$\dot{\hat{d}}_1 = -\frac{1}{m_1}\Gamma_1 B^\top P_1 x_1 - \frac{n}{m_1}\Gamma_1 B^\top (P_2 + P_3)(x_1 - x_{\mathrm{ave}}), \tag{41}$$

$$\begin{aligned}\dot{\hat{d}}_i = &-\frac{1}{m_i}\Gamma_i B^\top P_1 x_i - \frac{1}{m_i}\Gamma_i B^\top P_2 (x_i - x_1) \\ &-\frac{n}{m_i}\Gamma_i B^\top P_3 (x_i - x_{\mathrm{ave}}), \qquad i \in \{2, 3, \dots, n\}.\end{aligned} \tag{42}$$

This is the cooperative disturbance estimation law.

We investigate the convergence of x in this case. From Young's inequality, the fourth term in (39) becomes

$$\begin{aligned}&2w^\top (M^{-1} \otimes B^\top)Px \\ &= 2w^\top (M^{-1} \otimes I_3) R^{\frac{1}{2}} R^{-\frac{1}{2}} (I_n \otimes B)^\top P x \\ &\leq w^\top (M^{-1} \otimes I_3) R (M^{-1} \otimes I_3) w \\ &\quad + x^\top P (I_n \otimes B) R^{-1} (I_n \otimes B)^\top P x.\end{aligned} \tag{43}$$

Using (40) and (43), the right-hand side of (39) can be evaluated as

$$\begin{aligned}\dot{V}(x) &\leq -x^\top C^\top Q C x + w^\top (M^{-1} \otimes I_3) R (M^{-1} \otimes I_3) w \\ &\leq -x^\top C^\top Q C x + \sigma_{\max}(R_{\mathrm{id}}) w_0^2 \sum_{i=1}^{n} \frac{1}{m_i^2}.\end{aligned} \tag{44}$$

Hence, we can see from a Lyapunov-like theorem for the ultimate-boundedness [17] that, under the control law (29) and the update law (40), the state x converges to the set

$$\left\{ x \in \mathbb{R}^{6n} \,\middle|\, x^\top C^\top Q C x \leq \sigma_{\max}(R_{\mathrm{id}}) w_0^2 \sum_{i=1}^{n} \frac{1}{m_i^2} \right\}.$$

The size of this set can be adjusted by using the weights Q and R_{id}. If R_{id} is chosen to be small, the right-hand side becomes smaller and x converges to a narrower region. However, this reduces the weights of the control inputs and might make it difficult for the system to achieve the desired behavior.

IV. ATTITUDE CONTROL

In this section, we construct a control law and an update law for torque disturbance estimation in rotational motion. The control law is designed by following [15]. The update law, which estimates the steady term of the torque disturbance is formulated based on Lyapunov stability theory.

A. Attitude control law

The attitude control law is designed so that the attitude R_i tracks the reference attitude R_i^{d} given in (16) and that the angular velocity ω_i tracks a reference angular velocity $\omega_i^{\mathrm{d}}(t) \in \mathbb{R}^3$, which is obtained by the attitude kinematics equation $\omega_i^{\mathrm{d}} = (-\dot{R}_i^{\mathrm{d}} R_i^{\mathrm{d}\top})^\vee$. The reference attitude R_i^{d} is constructed based on the virtual input v_i, which is derived from cooperatively formulated input u. Therefore, the resulting control law becomes a cooperative one. To construct such a control law, an attitude tracking error vector and an angular velocity tracking error vector are introduced.

The attitude tracking error is obtained from the time derivative of Ψ defined in (17). The time derivative of Ψ_i is given by

$$\dot{\Psi}_i = \frac{1}{2}\big(\omega_i - \omega_i^{\mathrm{d}}\big)^\top \big(R_i^{\mathrm{d}} R_i^\top - R_i R_i^{\mathrm{d}\top}\big)^\vee = \big(\omega_i - \omega_i^{\mathrm{d}}\big)^\top e_{R_i}, \tag{45}$$

where $e_{R_i}(t) \in \mathbb{R}^3$ is the attitude tracking error defined by

$$e_{R_i}(t) = \frac{1}{2}\big(R_i^{\mathrm{d}}(t) R_i(t)^\top - R_i(t) R_i^{\mathrm{d}}(t)^\top\big)^\vee. \tag{46}$$

The tracking error for the angular velocity $e_{\omega_i} \in \mathbb{R}^3$ is defined as follows:

$$e_{\omega_i}(t) = \omega_i(t) - \omega_i^{\mathrm{d}}(t) + K_{R_i} e_{R_i}(t), \tag{47}$$

where $K_{R_i} \in \mathbb{R}^{3\times 3}$ is a positive-definite design parameter. The third term $K_{R_i} e_{R_i}$ on the right-hand side of (47) is newly introduced in this paper to guarantee the convergence of the attitude directly.

The control objective for attitude motion shown in (18) is achieved if the following relation holds true:

$$\lim_{t\to\infty} e_{\omega_i}(t) = 0,\ \lim_{t\to\infty} e_{R_i}(t) = 0, \tag{48}$$

Actually, when the torque disturbance estimation is accomplished completely, that is, $\hat{d}_{ti} = d_{ti}$, the following torque input results in (48):

$$\begin{aligned}\tau_i =& -J_i k_{R_i} e_{R_i} - J_i K_{\omega_i} e_{\omega_i} + J_i\left(\dot{\omega}_i^{\mathrm{d}} - K_{R_i}\dot{e}_{R_i}\right)\\ &+ \omega_i \times (J_i\omega_i) - \hat{d}_{ti},\end{aligned} \tag{49}$$

where $k_{R_i} > 0$ and a positive-definite $K_{\omega_i} = K_{\omega_i}^\top \in \mathbb{R}^{3\times 3}$ are design parameters. The stability under (49) can be confirmed by setting $\hat{d}_{ti} = d_{ti}$ in the discussion in the next subsection.

B. Cooperative torque disturbance estimation

We design an update law for the torque disturbance estimation based on the Lyapunov stability theory. From (5), (7), (19), (47) and (49), the closed-loop system, including unknown torque disturbance, is obtained as

$$\dot{e}_{\omega_i} = -k_{R_i} e_{R_i} - K_{\omega_i} e_{\omega_i} + J_i^{-1}(\tilde{d}_{ti} + w_{ti}). \tag{50}$$

Let a Lyapunov function candidate V_{ti} be given by

$$V_{ti} = e_{\omega_i}^\top e_{\omega_i} + 2k_{R_i}\Psi_i + \tilde{d}_{ti}^\top \Gamma_{ti}^{-1}\tilde{d}_{ti}, \tag{51}$$

where a positive-definite $\Gamma_{ti} \in \mathbb{R}^{3\times 3}$ is a design parameter for torque disturbance estimation. Form (19), (45), (47) and (50),the time derivative of V_{ti} is given by

$$\begin{aligned}\dot{V}_{ti} =& 2e_{\omega_i}^\top \dot{e}_{\omega_i} + 2k_{R_i}\dot{\Psi}_i + 2\tilde{d}_{ti}^\top\Gamma_{ti}^{-1}\dot{\tilde{d}}_{ti}\\ =& -2e_{\omega_i}^\top K_{\omega_i} e_{\omega_i} - 2k_{R_i} e_{R_i}{}^\top K_{R_i} e_{R_i} + 2e_{\omega_i}^\top J_i^{-1} w_{ti}\\ &+ 2\tilde{d}_{ti}^\top (J_i^{-1} e_{\omega_i} - \Gamma_{ti}^{-1}\dot{\hat{d}}_{ti}).\end{aligned} \tag{52}$$

The first and second terms are negative definite. However, the remaining terms is indefinite. The third term remains because it contains an unknown value w_{ti}. On the other hand, we can eliminate the fourth term. Hence, the update law for torque disturbance estimation in rotational motion is formulated as

$$\dot{\hat{d}}_{ti} = \Gamma_{ti} J_i^{-1} e_{\omega_i}. \tag{53}$$

We next determine the convergence of the error variables e_{ω_i} and e_{R_i} in this case. From Young's inequality, the fourth term in (52) becomes

$$\begin{aligned}2e_{\omega_i}^\top J_i^{-1} w_{ti} =& 2e_{\omega_i}^\top K_{\omega_i}^{\frac{1}{2}} K_{\omega_i}^{-\frac{1}{2}} J_i^{-1} w_{ti}\\ \le& e_{\omega_i}^\top K_{\omega_i} e_{\omega_i} + w_{ti}^\top J_i^{-1} K_{\omega_i}^{-1} J_i^{-1} w_{ti}.\end{aligned} \tag{54}$$

Using (53) and (54), we can estimate (52) as

$$\begin{aligned}\dot{V}_{ti} \le& -e_{\omega_i}^\top K_{\omega_i} e_{\omega_i} - 2k_{R_i} e_{R_i}{}^\top K_{R_i} e_{R_i}\\ &+ w_{ti}^\top J_i^{-1} K_{\omega_i}^{-1} J_i^{-1} w_{ti}\\ \le& -e_{\omega_i}^\top K_{\omega_i} e_{\omega_i} - 2k_{R_i} e_{R_i}{}^\top K_{R_i} e_{R_i}\\ &+ \sigma_{\min}(J_i K_{\omega_i} J_i) w_{t0}^2.\end{aligned} \tag{55}$$

A Lyapunov-like theorem for the ultimate-boundedness [17] shows that the error variables e_{ω_i} and e_{R_i} converge to the set

$$\left\{(e_{\omega_i}, e_{R_i}) \,\middle|\, e_{\omega_i}^\top K_{\omega_i} e_{\omega_i} + 2k_{R_i} e_{R_i}{}^\top K_{R_i} e_{R_i} \le \sigma_{\min}(J_i K_{\omega_i} J_i) w_{t0}^2\right\}.$$

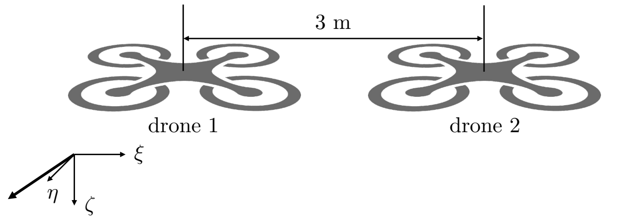

Fig. 3. The initial state of two drones and the wind vector occurring one second later.

The size of this set can be adjusted by tuning the design parameters K_{ω_i}, k_{R_i}, and K_{R_i}. If these parameters are chosen to be large values, in steady state, the error variables e_{ω_i} and e_{R_i} converge to a narrow region. However, in transient state, this may cause overshoot.

V. NUMERICAL SIMULATION

In this section, the validity of the update law for torque disturbance estimation is verified through numerical simulation. First, we confirm the achievement of the control objectives (10) and (18). Next, we evaluate the disturbance estimation performance (12) and (20). Finally, a necessity of the torque disturbance estimation is investigated by comparing the results with and without torque disturbance estimation.

The number of drones is set as $n = 2$, the mass is given by $m_1 = m_2 = 2$ [kg], and the moment of inertia is $J_1 = J_2 = \mathrm{diag}(0.01, 0.01, 0.02)$ [kg/m^2]. We assume that two drones hover 3 meters apart in the ξ-direction. The disturbance occurs after $t = 1$ [s]. The disturbance affecting translational motion is assumed to occur similarly. The steady terms of the translational disturbance are set as $\bar{w}_i = (0, 4, -1)^\top$, $i \in \{1, 2\}$, whereas the steady terms of the torque disturbance are such that $\bar{w}_{t1} = (4, 3, -1)^\top$ and $w_{t2} = (5, 4, 0)^\top$. The unsteady terms w_i and w_{ti} are generated by applying a first-order low-pass filter with cut-off frequency 1 to white noise with variance 1. We choose the weights in the cost function as $Q = I_{12}$ and $R = 0.05I_6$. The controller parameters for rotational motion are chosen as $k_{R_i} = 10, K_{R_i} = 100I_3$, and $K_{\omega_i} = 300I_3$ for each $i \in \{1, 2\}$. The design parameters in update laws for disturbance estimates (40) and (53) are given as $\Gamma_i = \mathrm{diag}(100, 100, 100)$, and $\Gamma_{ti} = \mathrm{diag}(0.06, 0.06, 0.3)$ for all $i \in \{1, 2\}$. The desired direction b is set as $b(t) = (1, 0, 0)^\top \in \mathbb{R}^3$. The target positions of the drone from a certain point are such that $l_1 = (-1.5, 0, 0)^\top$ and $l_2 = (1.5, 0, 0)^\top$. The initial state of the drones is set as $(p_1^\top, \dot{p}_1^\top) = (-1.5, 0, 0, 0, 0, 0), (p_2^\top, \dot{p}_2^\top) = (1.5, 0, 0, 0, 0, 0), R_1 = R_2 = I_3, \omega_1 = (0, 0, 0)^\top$, and $\omega_2 = (0, 0, 0)^\top$. The initial values of disturbance estimation are set as $\hat{d}_1 = \hat{d}_2 = (0, 0, 0)^\top$, and $\hat{d}_{t1} = \hat{d}_{t2} = (0, 0, 0)^\top$.

The velocities of the two drones are shown in Figs. 4 and 5. Their trajectories are shown in Fig. 6, which also shows the final relative distance between them. The attitude errors Ψ_1, Ψ_2 are shown in Fig. 7. These results indicate that control objectives are achieved. Two types of disturbance estimation when torque disturbance is considered are shown in Figs. 8–11. As can be seen from these results, the disturbance estima-

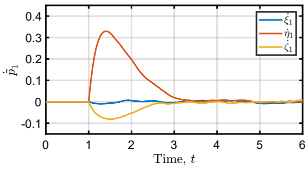

Fig. 4. The velocity of drone 1.

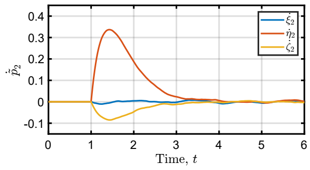

Fig. 5. The velocity of drone 2.

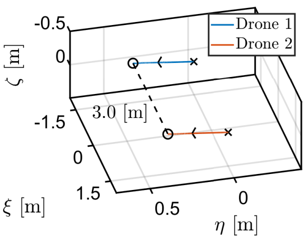

Fig. 6. The trajectories of two drones. The symbol × indicates the initial positions, while ∘ indicates the final positions.

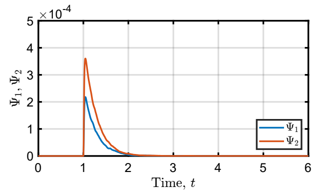

Fig. 7. The attitude error of two drones.

tion is achieved. The disturbance estimation for translational motion without considering torque disturbance is shown in Figs. 12 and 13. Comparing Figs. 8 and 9 with Figs. 12 and 13, it can be observed that the disturbance estimation for translational motion are achieved more accurately by incorporating torque disturbance estimation.

VI. CONCLUSIONS

In this paper, we designed a cooperative update law for torque disturbance estimation for multiple quadrotor drones. The disturbance acting on translational motion and the torque disturbance acting on rotational motion were simultaneously estimated and compensated. The effectiveness of this method is demonstrated through numerical simulations.

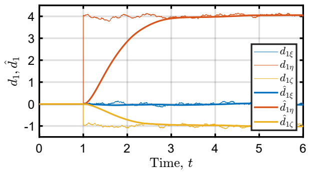

Fig. 8. True and estimated values of the disturbance for translational motion of Drone 1 when torque disturbance is considered.

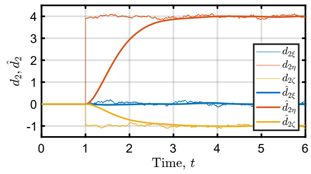

Fig. 9. True and estimated values of the disturbance for translational motion of Drone 2 when torque disturbance is considered.

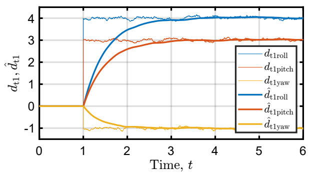

Fig. 10. True and estimated values of the torque disturbance for Drone 1.

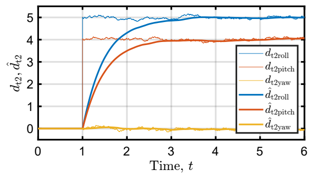

Fig. 11. True and estimated values of the torque disturbance for Drone 2.

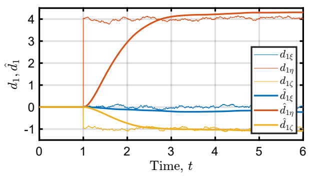

Fig. 12. True and estimated values of disturbance for translational motion of Drone 1 when torque disturbance is not considered.

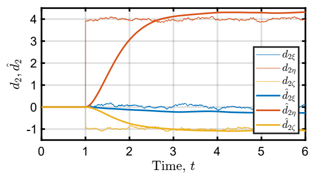

Fig. 13. True and estimated values of the disturbance for translational motion of Drone 2 when torque disturbance is not considered.

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